1.2 Dihedral groups (page 23)

Monday, October 9, 2023 13:20

D_{2n}

r

$D_{2n} = \{ r, s r \mid r^n = s^2 = 1, \; rs = s^{-1} r \}$

$n \in \mathbb{Z}$

if z is a power of r

then $z = r^k$ $k \in \mathbb{Z}$

take

$s = r^1$

then

$r^2 = 1$

so $n = 2$

$rs = r^2 = s^{-1} r = 1$

so $s^{-1} = 1 = r$

$1^{a} = 1^2 = 1^1$

the order is 1 for

$D_2 := \{1, s\}$

$r^1 = 1$

$\boxed{D_2 := \{r, s\}}$

the [illegible]

but $s = r^{i} \Rightarrow s = 37$
which is a power of r

lets see
$s \neq 37$

$\Rightarrow$

$s^2 = r^n$

$\Rightarrow \sqrt[2]{r^n} = s$

$\Rightarrow r^{\frac{n}{2}} = s$

for $r^k = 1$

$k = \frac{n}{2} \Rightarrow 2k$

so the order

then
$$s = \begin{cases} 0 \\ 1 \end{cases}$$

Suppose $s = 0$
the identity must show

$e\circ s = s = s\circ e$

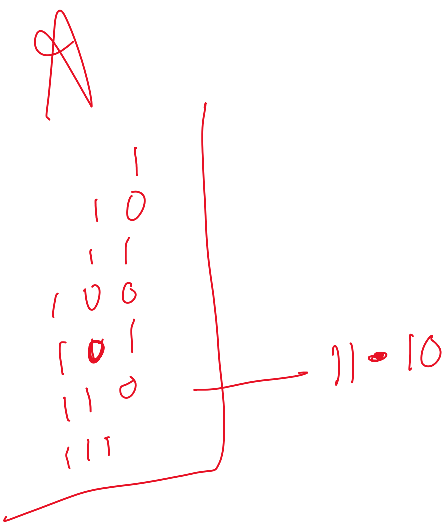
$\longrightarrow$ true

$e\circ r = r = r\circ e$

$s \neq r \Longrightarrow e\circ(rs) = (rs)\circ e = e$

$\not\Rightarrow e\circ(rs)s^{-1} = (rs)e s^{-1}$

$e\circ r = r$



if a number is not a power of r then the number s^2 cannot be considered neither can s^{2k} $k\in\mathbb{Z}$

Since $s^2 \circ s^2 \cdots\cdots s^{2(k-2)} = s^k$

$s^n \quad s^n \quad s^{n(k-2)}$

$= r^{\cdot} \circ r^{\cdot} \cdots$

So $s^1, s^3, \ldots, s^{(2k+1)}$ are good candites

take s^1

if $s^1 = e$ contradicts the premise since $r^n = e$

$\therefore s^1 \neq e$

s^1, sr, r^2, s^2, sr^3

$D_{2n} =$

if s^2 is a power of r then so is every s^{2k} $k \in \mathbb{Z}$

notice

$$rs = sr^{-1}$$

$$\Rightarrow$$

$$(rs)(sr^{-1}) = e$$

since $r(s^2)r^{-1}$

$$= (r \circ e) r^{-1}$$

$$= r \circ r^{-1} = e$$

$$e \circ s = s = s \circ e$$

$$r^n \circ s = s^2 \circ s = e \circ s$$ D_{12}

$$= s^3 = s$$

notice $k \in \mathbb{Z}$ $(k+1) \leq n$

$$r^{(k+1)} \circ s \Rightarrow s$$

if $x = r^{(k+1)} \circ s$

then

$$x^1 = s$$

$$x^2 = s^2 = e$$

thus 2 is the order of the D_{2n} group.