Lec. 25-d-core lemma

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The key is the following inequality.

Core Lemma:

$\forall \varepsilon > 0$ and $\forall P \in \mathcal{P}_{[a,b]}$

$$\overline{U}(P,f) - L(P,f) \geq \varepsilon \overline{U}\left(\chi_{Disc_\varepsilon(f)}\right)$$

recall

$\chi_{Disc_\varepsilon(f)}$ "the characteristic / indicator function of $Disc_\varepsilon(f)$"

Defn: let $S \subseteq [a,b]$ the outer Jordan Content of S denoted $\tau(S)$

$$\tau(S) := \overline{U}(\chi_S)$$

~ The upper Darboux Integral of the characteristic function of S

" S together with all of its limit points"

Core lemma $\Rightarrow$ this direction of Lebesgue thm

Assume $f \in \mathcal{R}([a,b])$

then we know that $\exists$ sequences

$P_m \in \mathcal{P}_{[a,b]}$ s.t.

$$\lim_{m \to \infty} \left(\overline{U}(P_m, f) - L(P_m, f)\right) = 0$$

plug in P_m in l.h.s. of core inequality to

get $\overline{U}(P_m, f) - L(P_m, f) \geq \varepsilon \tau(Disc_\varepsilon(f))$

$\therefore \tau(Disc_\varepsilon(f)) = 0 \quad \forall \varepsilon > 0$

Proposition: $\tau(Disc_\varepsilon(f)) = 0 \quad \forall \varepsilon > 0$

$\xLeftrightarrow{\text{iff}} |Disc(f)| = 0$