

# Lec. 25-c-Theorem H Lebesgue

Thursday, August 8, 2024

8:28 AM

Let  $f: [a, b] \rightarrow \mathbb{R}$  be a bounded function

then  $f \in \mathcal{R}([a, b]) = \mathcal{D}([a, b])$  iff  $|\text{Disc}(f)| = 0$

which is to say.

A bounded function  $f$  is continuous iff

$f$  is continuous almost everywhere

$f: [a, b] \xrightarrow{\text{cont}} \mathbb{R}$  is  $\in \mathcal{R}$

$\xleftrightarrow{\text{iff}}$   $f$  is  $\mathcal{C}^0$  almost everywhere

i.e. Cantor Statement

if  $f \in \mathcal{R}([a, b]) \Rightarrow |\text{Disc}(f)| = 0$

$|\text{Disc}(f)| = 0 \not\Rightarrow f \in \mathcal{R}([a, b])$