Lec. 25-b-claims

Thursday, August 8, 2024 8:16 AM

finite

$\# |A|$

claim:

let $S \subseteq [a,b]$

assume S is finite or countable

then $|S| = 0$

part II

let $f : [a,b] \to \mathbb{R}$

let $x_0 \in [a,b]$

The oscillation of f @ x_0

denoted by $osc(f)(x_0)$ – is

$$osc(f)(x_0) := \lim_{\delta \downarrow 0} \sup \left\{ |f(u) - f(v)| \;\middle|\; u,v \in (-\delta + x_0, x_0 + \delta) \cap [a,b] \right\}$$

claim

$f : [a,b] \to \mathbb{R}$

let $x_0 \in [a,b]$ then f is discontinuous with x_0

$\xleftrightarrow{\text{iff}}$ $osc(f)(x_0) > 0$

Next: "the ε discontinuity of f:"

let $\varepsilon > 0$ $Disc_\varepsilon(f) := \{ x \in [a,b] \mid osc(f)(x) \geq \varepsilon \}$

claim $Disc_\varepsilon(f)$ is closed inside $[a,b]$

$\therefore$ $Disc_\varepsilon(f)$ is compact

Def^n $Disc(f) = \bigcup_{\varepsilon > 0} Disc_\varepsilon(f)$

claim if $0 < \alpha < \beta$

then $Disc_\beta(f) \subseteq Disc_\alpha(f)$

corollary $Disc(f) = \bigcup_{n \geq 1} Disc_{\frac{1}{n}}(f)$

the total discontinuity is a countable union of compact sets