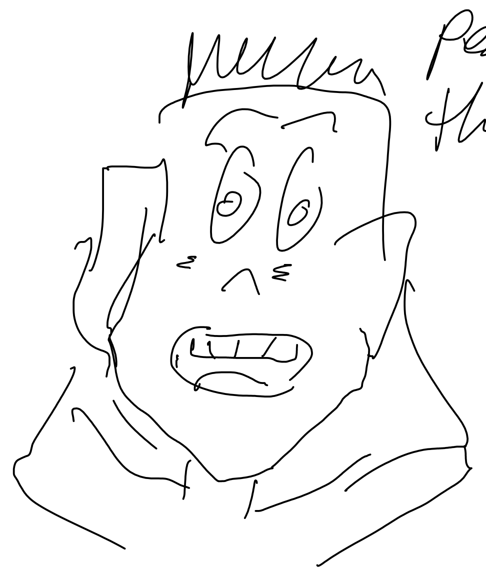
Lec. 24-c- Definitions, characteristics indicator function

Wednesday, July 31, 2024 8:02 PM

Defn let S be any set $(\neq \emptyset)$
let $A \subseteq S$ $(A \neq \emptyset)$
The characteristic function χ_A of A
is the function

$$\chi_A : S \longrightarrow \{0,1\}$$


People in the 1800s

$$\chi_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}$$

or $\chi_A = \mathbb{1}_A$ "indicator form"

(9) let $a<b$, let $f = \chi_{\mathbb{Q} \cap [a,b]}$
then $f \notin \mathcal{R}([a,b])$

~~proof~~ $\forall P \in \mathcal{P}_I$ $L(P,f) = 0$ & $U(P,f) = 1$
.... can't be less than $\forall \varepsilon > 0$
"it's discontinuous at every point in the interval"

(10) let $\{r_1, r_2, \ldots r_n\}$ be an enumeration of the rationals in $[a,b]$

let $f_n := \chi_{\{r_1, r_2, \ldots \ldots r_n\}}$

then $f_n \in \mathcal{R}([a,b])$ all n (w/) $\int_a^b f_n = 0$
& $f_n \longrightarrow \chi_{\mathbb{Q} \cap [a,b]}$ "it is pointwise convergent but pointwise limit is not integrable"

main point: $f_n \in \mathcal{R}$ but, $\lim_{n \to \infty} f_n \notin \mathcal{R}$

(11) $f \in \mathcal{R}([a,b])$ & $c \in (a,b)$
then $f \in \mathcal{R}([a,c]) \cap \mathcal{R}([c,b])$
moreover, $\int_a^b f = \int_a^c f + \int_c^b f$

(12) $f \in C^0([a,b])$ & $f \geq 0$
then $\int_a^b f \geq 0$ & $= 0$ iff $f = 0$

Next time on Dragonball Z
"Big finish"

Lebesgue's characterization of Riemann-Darboux integrability

$f \in \mathcal{R}([a,b])$ iff $|\mathrm{disc}(f)| = 0$

$| \quad |$ = "measure"