Lec. 24-b- lipschitz

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Recall Lipschitz from ordinary diff. equations

$$\left|\Phi(x)-\Phi(y)\right| \le R\,|x-y|$$

⑤ let $f \in \mathcal{R}([a,b])$ w/ $|f| \le M$ on $[a,b]$

let $\Phi \in C^0_{Lip}([-M,M])$ — similar to Problem 4

Then $\Phi \circ f \in \mathcal{R}([a,b])$ — ~~try~~ this ~~$\Rightarrow$~~

⑥ $f, g \in \mathcal{R}([a,b])$

then $f \cdot g \in \mathcal{R}([a,b])$

Hint $f(x)g(x) = \frac{1}{2}\left(f(x)+g(x)\right)^2 - \frac{1}{2}f(x)^2 - \frac{1}{2}g(x)^2$

⑦ let $f:[a,b] \to \mathbb{R}$ bounded.

& assume $f \in C^0((a,b]) \Rightarrow f \in \mathcal{R}([a,b])$

Closed on lower or upper — $X \subseteq \mathbb{R}_l, \mathbb{R}_u$

Main Point.

"a single (bounded) discontinuous of f still provides integrability"

$\mathcal{R}$ Correlated

Defn: let $f: [a,b) \xrightarrow{\text{bounded}} \mathbb{R}$ (closed) then the discontinuity on f

$$\operatorname{disc}(f) := \{x \in [a,b] \mid f \text{ is discontinuous at } x\}$$

⑧ let $f:[a,b) \xrightarrow{\text{bnded}} \mathbb{R}$

assume $\operatorname{disc}(f)$ is finite $(\#\operatorname{disc}(f) < \infty)$

then $f \in \mathcal{R}([a,b])$