Lec. 24 - Examples and Exercises

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Pages 152-154

last time $\mathcal{D}([a,b]) = \mathcal{R}([a,b])$
all with the same value of the integral

Corollary: If $f \in \mathcal{D}([a,b])$ & P_n is the n^{th} "standard partition" of $[a,b]$
i.e. $x_i = a + i\left(\frac{b-a}{n}\right)$

then $\lim_{n\to\infty} R((P_n, \{x_i\}) : f) = \int_a^b f = U = L$

think

Pros		Cons	
$\mathcal{D}$	$\mathcal{R}$	$\mathcal{D}$	$\mathcal{R}$

Prove:
1.) F.T.C.
(I) let $f \in C^0([a,b])$
- assume $f'(x)$ $\forall x \in (a,b)$
- assume $f' \in \mathcal{R}([a,b])$

Note: $\exists$ differentiable functions f with non-integrable derivatives
i.e. - Volterra's non-integrable derivatives

Prove me

But if $f \in C^0([a,b])$, and $f'(x)$ $\forall x \in (a,b)$ for $f' \in \mathcal{R}([a,b])$

then $\int_a^b f' = f(b) - f(a)$.

MEAN VALUE THEOREM and TELESCOPING SUM

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(2). F.T.C. (II)
let $f \in C^0([a,b])$
Define $F(x) := \int_a^x f$ then

a) F' exists for all $x \in (a,b)$
b) $F'(x) = f(x)$