

# Lec. 23-b-TIE

Wednesday, July 31, 2024 4:50 AM

$$|R(P, \underline{\xi}) - L(P)| < \frac{\epsilon}{4}$$

and

$$|R(P, \bar{\xi}) - U(P)| < \frac{\epsilon}{4}$$

so, by the Triangle inequality, we get

$$|U(P) - L(P)| = |U(P) - R(\bar{\xi}) + R(\bar{\xi}) - R(\underline{\xi}) + R(\underline{\xi}) - L(P)|$$

$$\leq |U(P) - R(\bar{\xi})| + |R(\bar{\xi}) - R(\underline{\xi})| + |R(\underline{\xi}) - L(P)|$$

$$\underbrace{\hspace{10em}}_{< \frac{\epsilon}{4}} \quad \underbrace{\hspace{10em}}_{< \frac{\epsilon}{2}} \quad \underbrace{\hspace{10em}}_{\frac{\epsilon}{4}}$$

$$\Rightarrow \text{R.H.S.} < \epsilon$$

this shows  $f \in \mathcal{R} \Rightarrow f \in \mathcal{J}$ .