Lec. 23-a- Main Theorem (easy direction)

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Main Theorem: $\mathcal{D}([a,b]) = \mathcal{R}([a,b])$

$\Rightarrow$ $f \in \mathcal{R} \Rightarrow f \in \mathcal{D}$

Since $f \in \mathcal{R}$ we have by definition of Rieman Integrability

$$\forall \varepsilon > 0 \ \exists \delta(\varepsilon) > 0 \text{ s.t. } |R(P, \xi) - I| < \frac{\varepsilon}{4}$$

"the Reiman Sum of the marked partition" $\quad I(f)$

"for any marked partition (P, ξ) w/ $\|P\| < \delta(\varepsilon)$

$\therefore$ for any two markings $(P, \overline{\xi})$ & $(P, \underline{\xi})$ we obtain the following inequality

$$|R(P, \overline{\xi}) - R(P, \underline{\xi})| < \frac{\varepsilon}{2}$$

Fix this partion P "upper sum relative to said P minus lower sum relative to P is less than $\varepsilon/2$" by T.I.E.

Claim:

$$U(P,f) - L(P,f) < \varepsilon$$

"this shows that $f \in \mathcal{D}([a,b])$" hence f is Darboux integrable

Observe

given $\varepsilon > 0$ we have

a.) $\exists$ marking $\underline{\xi}$ s.t.

$$L(P,f) \leq R(P, \underline{\xi}, f) < L(P,f) + \varepsilon/4$$

b.) $\exists$ marking $\overline{\xi}$ s.t.

$$U(P,f) - \varepsilon/4 \leq R(P, \overline{\xi}, f) \leq U(P,f)$$

why they hold

Comes directly from the definition

Sup and inf

also came from definitions but more nuanced...

Cake explanation

... they arrive by the idea of displacement
add a small amount to greatest lower bound
subtract a little bit from the least upper bound.

next, put it together in T.I.E.