Lec. 22-f- Riemann Integral definition

Tuesday, July 30, 2024 2:21 AM

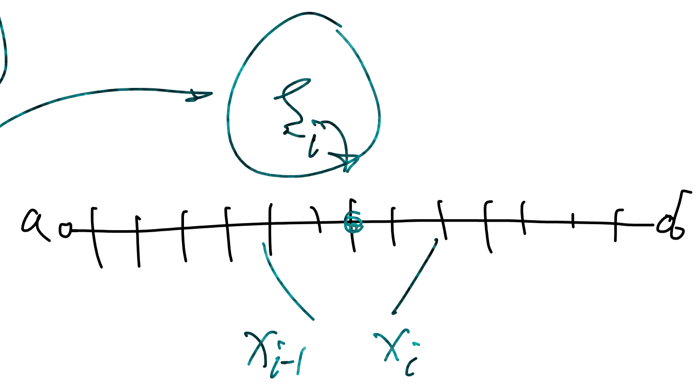
let $P \in \mathcal{P}_I$, $I = [a,b]$

$P = \{a = x_0 < x_1 \cdots < x_n = b\}$ or some other partition

a "marking" or "tagging" of P is a choice ξ_i

$\xi_i = (\xi_1, \xi_2 \dots \xi_n)$

$\xi_i \in [x_{i-1}, x_i]$



$\Uparrow$ ith "bin"

Given $f : [a,b] \to \mathbb{R}$ and any tagged ("sample") partition (P, ξ_i)

The Riemann Sum of f relative to (P, ξ_i) is defined by:

$$R((P; \xi_i); f) := \sum_{i=1}^{n} f(\xi_i)(x_i - x_{i-1})$$

Defn: $f : [a,b] \to \mathbb{R}$ is Riemann integrable

$\xleftrightarrow{\text{iff}}$ $\exists$ Real # $I(f)$ satisfying:

$\forall \varepsilon > 0$ $\exists \delta(\varepsilon) > 0$ s.t. all tagged partitions

$(P; \xi_i)$ w/ $\|P\| < \delta(\varepsilon)$

"mesh size" $\|P\| = \min\{x_i - x_{i-1}\}$

$$\Rightarrow \underbrace{|R - I(f)|}_{\text{get this close to Epsilon}} < \varepsilon$$

by making $\delta(\varepsilon)$ & partition size very slender so that Epsilon can be a small tolerance on the Riemann Sum minus the integral.

* TRY to show Riemann Integrable $\Rightarrow C^0, [a,b]$