Lec. 22-c- example 2

Tuesday, July 30, 2024 1:39 AM

Let $a=0$ & $b \le \pi/2$ (smallest positive zero of $\cos(x)$)

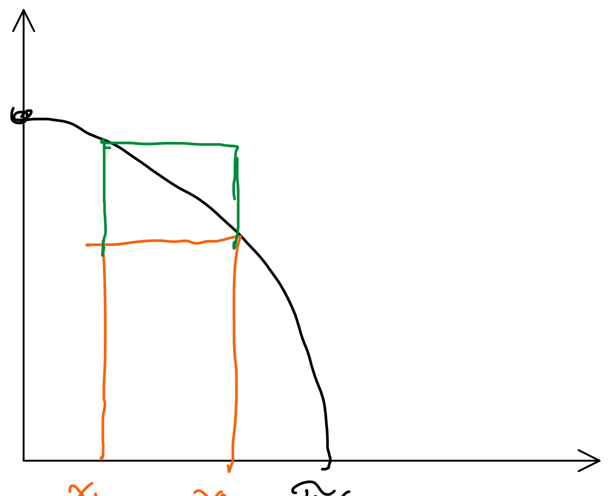
let $f(x) = \cos(x)$

$P_n = \{0 = x_0 < x_1 < \cdots < x_n = b\}$

$x_j = j\left(\frac{b}{n}\right)$. Let's compute $L(P_n; f)$ & $U(P_n; f)$

$$\boxed{L(P_n; \cos) = \sum_{j=1}^{n} \cos\left(j\left(\tfrac{b}{n}\right)\right)\frac{b}{n}}$$

$$\boxed{U(P_n; \cos) = \sum_{j=1}^{n-1} \cos\left(j\left(\tfrac{b}{n}\right)\right)\frac{b}{n}}$$



we need to know how to sum:

$\cos(\theta) + \cos(2\theta) + \cos(3\theta) + \cdots + \cos(n\theta)$.

step 1.

$\sin(A+B) = \cos(A)\sin(B) + \cos(B)\sin(A)$

step 2.

$\sin(A-B) = -\cos(A)\sin(B) + \cos(B)\sin(A)$

step 3.

$\sin(A+B) - \sin(A-B) = 2\cos(A)\sin(B)$

let $A = j\theta$ & $B = \frac{\theta}{2}$ in 3

obtain:

$$\sin\left((j+1)\theta - \theta/2\right) - \sin\left(j\theta - \theta/2\right) = 2\cos(j\theta)\sin\left(\tfrac{\theta}{2}\right)$$

$$\Longleftrightarrow \quad \cos(j\theta) = \frac{\sin\left((j+1)\theta - \theta/2\right) - \sin\left(j\theta - \theta/2\right)}{2\sin\left(\theta/2\right) \neq 0}$$

positive

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we can now compute

$$L(P_n, \cos) = \frac{\sin\left((n+\frac{1}{2})\frac{b}{n}\right) - \sin\left(\frac{b}{2n}\right)}{2\sin\left(\frac{b}{2n}\right)} * \left(\frac{b}{n}\right)$$

#\# study $\lim_{n\to\infty}$ of $L(P_n; \cos) = \boxed{\sin(b)}$

expected

try $U(P_n; \cos)$

"find explicit formulas in the inequality"

note: Lower sum $j=1$ to n

Upper sum $j=0$ to $n-1$

$\Uparrow$ telescoping series

Could also use complex numbers (imaginary + reals)

$\Downarrow$ # try Geometric formula

compute

$$\sum_{i=1}^{n} (e^{i\theta})^n$$