Lec. 21-c Fundamental definitions

Monday, July 29, 2024 11:46 PM

Definition:

A bounded function $f : [a,b] \to \mathbb{R}$ is "Riemann - Darboux integrable"

$\xleftrightarrow{\text{Def}}$ $L([a,b]; f) = U([a,b]; f)$

denoted by "Supremum" - small, "infimum" - large of sums : "If they're equal then they're Riemann - integrable"

$\int_a^b f$ or $\int_a^b f(x)\,dx$

Notation:

$\mathcal{R}([a,b]) := \{ f : [a,b] \to \mathbb{R} \text{ bounded} \mid \int_a^b f \text{ exists} \}$

Theorem: T.F.A.E.

1. $f \in \mathcal{R}([a,b])$
2. $\forall \varepsilon > 0 \ \exists \ P(\varepsilon) \in \mathcal{P}_I$ s.t. $\underbrace{U(P(\varepsilon), f) - L(P(\varepsilon), f)}_{\geq 0 \text{ by def}^n} < \varepsilon$
3. $\exists$ sequence of partitions $P_m \in \mathcal{P}_I$ s.t. $\lim_{m \to \infty} \left(U(P_m; f) - L(P_m; f) \right) = 0$

Proof: ② $\Rightarrow$ ③

Assume 2 holds.

we know that $\exists$ 3 partitions.

$P(\varepsilon)$, $P_L(\varepsilon)$, $P_U(\varepsilon)$ s.t.

a.) $|L(I; f) - L(P_L(\varepsilon); f)| < \frac{\varepsilon}{3}$

b.) $|U(I; f) - L(P_U(\varepsilon); f)| < \frac{\varepsilon}{3}$

c.) $|U(P(\varepsilon); f) - L(P(\varepsilon); f)| < \frac{\varepsilon}{3}$

Now, by Δ I. EQ

$$|L - U| \leq |L - L(P^*(\varepsilon))| + |L(P^*(\varepsilon)) - U(P^*(\varepsilon))| + |U - U(P^*(\varepsilon))| < \left(\frac{\varepsilon}{3}\right) \cdot 3$$

where $P^*(\varepsilon)$ is a common refinement of

$P(\varepsilon) \cup P_L(\varepsilon) \cup P_U(\varepsilon) =: P^*(\varepsilon)$

Hence

$\forall \varepsilon > 0$ we obtain $|L - U| < \varepsilon$ $\therefore$

$L = U$ & $f \in \mathcal{R}([a,b])$ ∎

Show $1 \Rightarrow 2$

Show $3 \Rightarrow 1$

Next time on Dragonball Z

. . . .

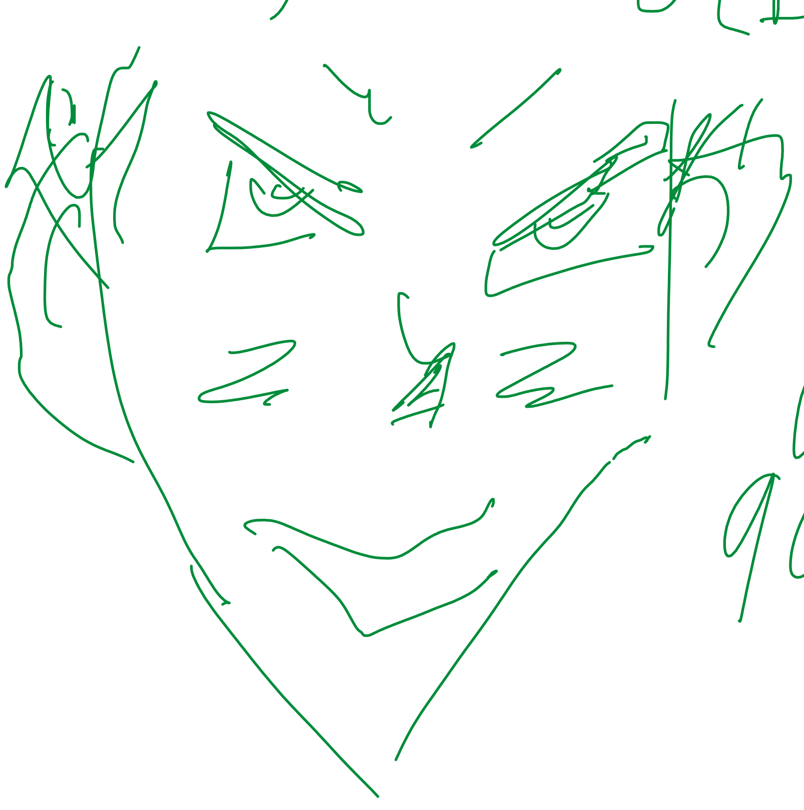
$\Downarrow$

$L(P; f)$ & $U(P; f)$ for $f(x) = x^2$

$= e^x$

$= \sin(x)$

$= \cos(x)$



over 9000!!!