Lec. 21-b proof, lower sum leq upper sum

Monday, July 29, 2024 11:41 PM

WTS $L(f) \leq U(f)$ is $\equiv$

$\forall\ P_1, P_2 \in \mathcal{P}_I$ we have $L(P_1, f) \leq U(P_2, f)$

recall: when $P_1 = P_2$ — the above inequality holds trivially.

Proof $P_1 \neq P_2$

let $P^* = P_1 \vee P_2$ "common refinement of P_1 & P_2"

Then

$$L(P_1; f) \leq L(P^*; f) \leq U(P^*; f) \leq U(P_2; f)$$

$\therefore L(f) \leq U(f)$ for all bounded $f : [a,b] \longrightarrow \mathbb{R}$