Lec. 20-b-Corollaries

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$$\lim_{h\to 0} \frac{(x+h)^n - x^n}{h} = n x^{n-1} \qquad (x^{n+1})' = n x^n$$

$\sin'(x) = \cos(x)$

$\cos'(x) = -\sin(x)$

$(e^x)' = e^x$

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$\sin(x+h) = \ldots$ product Thm.

$\cos(x)\sin(h) + \cos(h)\sin(x)$

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h} = \cos(x)\left(\frac{\sin(h)}{h}\right) + \sin(x)\left(\frac{\cos(h)-1}{h}\right)$$

Converges

$$\cos(x) + 0 \quad \square$$

the definition of sin and cos are uniform cauchy limit of a sequence of polynomials

Since we showed

$\sin'(x) = \cos(x)$ and $\cos'(x) = -\sin(x)$

$\Rightarrow \cos^2(x) + \sin^2(x) = 1$

Why?

let $f(x) = \cos^2(x) + \sin^2(x)$

$$f'(x) = 2\cos(x)\cdot\cos'(x) + 2\sin(x)\sin'(x)$$

$$= 2\cdot\cos(x)\cdot(-\sin(x)) + 2\cdot\cos(x)\cdot\sin(x) = 0$$

By M.V.T. $f \equiv$ Constant

But $\cos(0) = 1$ and $\sin(0) = 0$

$\therefore \cos^2(x) + \sin^2(x) \equiv 1$