Lec. 20-a-derivatives of trig func.

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Uniform Convergence shows that

$\sin(x)$, $\cos(x)$, e^x

$\overset{2}{S}(x)$

are all C^0 on any $[a,b]$, $a<b$

Claim: $\forall\, a<b$ $\sin(x)$, $\cos(x)$ and $e^x \in C^0([a,b])$

Keys: product Theorem.

Try this:

☆ a) $\lim_{h\to 0} \frac{\sin(h)}{h} = 1$

☆ b) $\lim_{h\to 0} \frac{\cos(h)-1}{h} = 0$

☆ c) $\lim_{h\to 0} \frac{e^h-1}{h} = 1$

Get ready to become a human calculator

$$P_n(x) := \sum_{m=0}^{n} (-1)^m \frac{x^{2m+1}}{(2m+1)!} \qquad 0<|x|<1$$

$P_n(x)$ is an alternating series:

$$\underbrace{\frac{|x|^{2m+3}}{(2m+3)!}}_{b_{m+1}} < \underbrace{\frac{|x|^{2m+1}}{(2m+1)!}}_{b_m}$$

$\therefore$ The A.S.T. Estimate

lec. 12

$$\left|\sin(h) - P_n(h)\right| \overset{*}{\leq} \frac{|h|^{2n+1}}{(2n+1)!}$$

Then

$$\left|\frac{S(h)}{h} - \frac{P_n(h)}{h}\right| \leq \frac{|h|^{2n}}{(2n+1)!} \longrightarrow 0$$

as $h \to 0$

$\top$ $h \neq 0$

but

$$\frac{P_n(h)}{h} = 1 + O(h^2) \longrightarrow 1$$

so

$$\left|\frac{S(h)}{h} - 1\right| \leq \left|\frac{S(h)}{h} - \frac{P_n(h)}{h}\right| + \left|\frac{P_n(h)}{h} - 1\right|$$

$$\text{AIEQ} \qquad \frac{|h|^{2n}}{(2n+1)!} \qquad \overset{||}{O(h^2)}$$