Lec. 19-d-Mean value theorem

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Rolle's Theorem $\Rightarrow$ MVT

MVT is a corollary to Rolle's Theorem.

$f \in C^0([a,b])$ assume $f'(x)$ exists all $x \in (a,b)$

Then $\exists\ c \in (a,b)$ s.t. $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

slope of tan?

slope of secant

Proof Define $\Phi_1(x)$

$$\Phi_1(x) := f(x) - \left(f(a) + \left(\frac{f(b)-f(a)}{b-a}\right)(x-a)\right)$$

Observe: $\Phi_1(a) = \Phi_1(b) = 0$

Differentiable since
f is diff.
Polynomials are differentiable

Prove this with difference quotient

$\therefore$ there is a point between a and b where the derivative vanishes.

So by Rolle's theorem $\exists\ c \in (a,b)$ St.

$\Phi_1'(c) = 0 \iff f'(c) = \dfrac{f(b)-f(a)}{b-a}$

Corollary: If $f'(x) = 0\ \forall x \in (a,b)$
then $f \equiv C$ a constant

Defn: $f : [a,b] \to \mathbb{R}$ we say f is increasing ($f \uparrow$) provided

$$a \leq \alpha \leq \beta \leq b \Rightarrow f(\alpha) \leq f(\beta)$$

Decreasing ($f \downarrow$) $a \leq \alpha \leq \beta \leq b \Rightarrow f(\alpha) \geq f(\beta)$

Hence Corollary of MVT:

f is $\uparrow$ iff $f'(x) \geq 0$

f is $\downarrow$ iff $f'(x) \leq 0$

for any differentiable function $f \in C^0([a,b])$