Lec. 19-c-Rolle'S theorem

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let $f \in C^0([a,b])$

assume $f(a) = f(b)$ ☆

If $f'(x)$ exists $\forall\, x \in (a,b)$

then $\exists\, c \in (a,b)$ s.t. $f'(c) = 0$

Proof By the max principle $\exists\, c_{min}, c_{max} \in [a,b]$

s.t. $f(x) \leq f(c_{max}) \;\forall x$

$f(c_{min}) \leq f(x) \;\forall x$

If <u>either</u> c_{min} or $c_{max} \in (a,b)$

$\Rightarrow f'(c_*) = 0$, f is constant $\therefore f'(x) = 0 \;\forall x \in (a,b)$

basically when $f(max) = f(min)$, f is constant.

the existence of c comes from this