Lec. 19-a-Rigorous calculus

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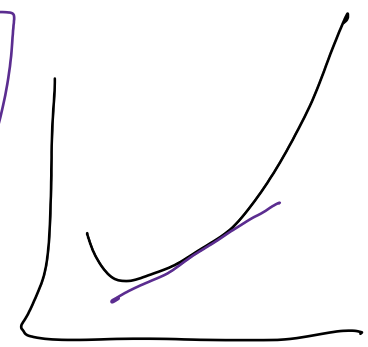
let $f:[a,b]\to\mathbb{R}$, let $x_0\in(a,b)$ "x_0 is an interior point"

"f is a a real valued function on an interval"

we say, "f is differentiable at x_0 provided the limit ~~exist~~"

$$\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}=:f'(x_0)$$

$$\boxed{f'(x_0) = \text{the derivative of } f \text{ at } x_0}$$



$$f'(x_0)=\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$$

$$f'(x_0)=\lim_{j\to+\infty}\frac{f(x_j)-f(x_0)}{x_j-x_0} \qquad x_j\to x_0 \text{ and } x_j\neq x_0$$

If $f'(x_0)$ exists then f is C^0 @ x_0

Proof use $\big||a|-|b|\big|\le|a-b|$ to get

$$|f(x_j)-f(x_0)|\le \frac{\varepsilon}{2}|x_j-x_0|+|f'(x_0)|\cdot|x_j-x_0|$$

$$=\left(\frac{\varepsilon}{2}+|f'(x_0)|\cdot|x_j-x_0|\right)$$