Lec. 18-3-Uniform continuance

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given

$$f \in C^0(X,d),\ f: X \xrightarrow{C^0} \mathbb{R}$$

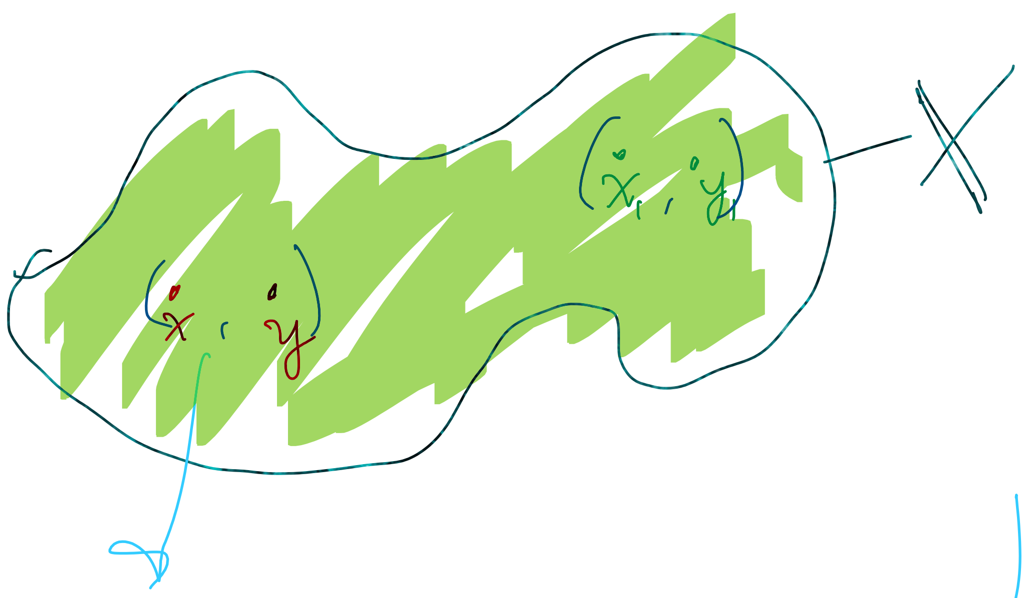
$\Rightarrow$ f is C^0 @ $\forall x_0 \in X$

$\Longleftrightarrow$

$\forall \varepsilon > 0 \quad \exists \delta = \delta(x_0, \varepsilon)$ s.t.

$$d(y, x_0) < \delta(x_0, \varepsilon) \Rightarrow |f(y) - f(x_0)| < \varepsilon$$

so the delta always depends on ε but here it also depends on x_0



- if x is close to y we can make an ε — $|f(y) - f(x)| < \varepsilon$ — b/c f is C^0
- if x_1 y_1 " " b/c f is C^0

$$|f(y_1) - f(x_1)| < \varepsilon_1$$

Q: is the $d(x, y) \stackrel{?}{=} d(x_1, y_1)$

no, the distances will differ

Uniform continuity says that the same delta can be used everywhere in X

$\Rightarrow$ Def^n

Given $f: (X, d) \xrightarrow{C^0} \mathbb{R}$ is uniformly C^0 provided that

$\forall \varepsilon > 0 \quad \exists \delta > 0$ s.t.

$$d(x, y) < \delta = (\delta(\varepsilon)) \Rightarrow |f(x) - f(y)| < \varepsilon$$

so that δ need not depend on the pairs of points. just epsilon

$\Leftrightarrow$

Theorem:

let (K, d) be a compact metric space
let $f \in C^0(K, d)$

$\Rightarrow$ f is uniformly C^0 ∎