L 17-4 Theorem def Continuity with bounded

Friday, July 19, 2024 11:43 AM

let (X, d) be a metric Space

Def^{n}: $C_B^0(X, d) = \{ f : X \to \mathbb{R} \mid \text{s.t. } \sup_{x \in X} (f(x)) < +\infty \}$

$\parallel$

All bounded

C^0 functions f on X.

may define $d_\infty(f, g) := \sup_{x \in X} (f(x) - g(x))$

then d_∞ is a metric

Theorem $\left(C_B^0(X, d), d_\infty(\cdot, \cdot) \right)$

is a complete metric space

Corollary:

$C^0([a,b]) = C_B^0([a,b])$

by theorem $C^0([a,b])$ is complete

$\therefore$ $B_{\delta_m}(y_0) \subseteq C^0([-\delta + t_0, t_0 + \delta])$

is also complete.