L 17-3-Cauchy like criterion

Friday, July 19, 2024 11:35 AM

let S be non-empty

Def^n:

$f_n : S \longrightarrow \mathbb{R}$ is cauchy

$\overset{\Longleftrightarrow}{\text{iff}}$ $\{f_n(x)\}$ is cauchy $\subseteq \mathbb{R}$

for all $x \in S$ given (f_n) cauchy

can define

$$f(x) := \lim_{n \to +\infty} f_n(x)$$

$\therefore$ $f_n : S \longrightarrow \mathbb{R}$ cauchy iff f_n converges to some $f : S \longrightarrow \mathbb{R}$

Def^n: $f_n : S \longrightarrow \mathbb{R}$ is uniformly cauchy

iff $\forall \varepsilon > 0 \ \exists N > 0 \in \mathbb{Z}_{>0}$ s.t. $\forall n, m \geq N$

$$\sup_{x \in S} \left(f_n(x) - f_m(x) \right) < \varepsilon$$

$N = N(\varepsilon)$ independent of $x \in S$

$\overset{\text{then}}{\Longrightarrow}$ A uniformly Cauchy sequence $f_n : S \longrightarrow \mathbb{R}$ converges uniformly to its pointwise limit

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