L 17-2-Ordinary differential equations

Thursday, July 18, 2024 10:43 AM

existance / uniqueness theorem.

Recall the setup.

a.) $\Omega \subset \mathbb{R}^2$ non-empty & open **Connected**

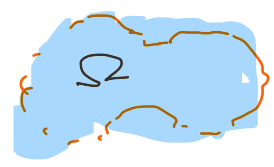
b.) $F: \Omega \xrightarrow{C^0} \mathbb{R}$

F is Lipshitz in the 2nd variable (i.e. the y)

$(t,y) \in \Omega \quad \exists \underset{\text{constant}}{K} > 0$

$|F(t,y_1) - F(t,y_2)| \leq K|y_1 - y_2|$

c.) fix some $(t_0, y_0) \in \Omega$



then $\Rightarrow \exists \delta > 0$ and a differentiable

$y: (-\delta + t_0, t_0 + \delta) \longrightarrow \mathbb{R}$

satisfying:

1.) $(t, y(t)) \in \Omega \quad |t - t_0| < \delta$

2) $y(t) = y_0 \quad (t_0, y_0) \in \Omega$

"initial condition"

3.) $y'(t) = F(t, y(t)) \quad \forall t \in (-\delta + t_0, t_0 + \delta)$