16-2-Intermediate value theorem

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let $f \in C^0([a,b])$

Then $f([a,b]) = [c,d]$

Note: $c \leq f(x) \leq d$ for all $x \in [a,b]$

since $c,d \in f([a,b])$ $\exists$ $x_{min} \in [a,b]$ @ $f(x_{min}) = c$

$x_{max} \in [a,b]$ @ $f(x_{max}) = d$

wlog: $f(a) < f(b)$

WTS.: $\forall y \in \mathbb{R}$ s.t. $f(a) < y < f(b)$

$\exists$ $x \in (a,b)$ s.t. $f(x) = y$.

Proof ingredients.

Lemma 1:

let $x_0 \in (a,b)$ & $t_0 \in \mathbb{R}$

let $g \in C^0([a,b])$ satisfies $g(x_0) < t_0$

Then $\exists$ $\delta > 0$ s.t. $g(x) < t_0$ $\forall$ $x \in (x_0 - \delta, \delta + x_0)$

Lemma 2

let $\{x_j\} \subseteq [a,b]$ (Cauchy $\therefore$ Convergent).

Assume $f(x_j) < t_0$ $\forall j$ where $f \in C^0([a,b])$

let $x_j \longrightarrow x_\infty$ then $f(x_\infty) \leq t_0$