Lec 14 - precise definition of a metric space

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Precise definition of a metric space:

- let X be a set $(\neq \{\})$
- let $d: X \times X \to \mathbb{R}$ satisfying:

a.) $d(p,q) = d(q,p)$

b.) $d(p,q) \geq 0 \ \forall p,q \in X$

Symmetric/reflexive and $=0$ iff $p=q$

c.) $d(p,z) \leq d(p,q) + d(q,z)$ triangle inequality

(X, d) is an abstract metric space

very core example $X = \mathbb{R}^N$
Key

d is the usual distance

Example:

let X be a continuous function on $[a,b]$ closed bounded

$:= \{f:[a,b] \to \mathbb{R}\}$ f is continuous

Define $d(f,g) := \sup\limits_{a \leq x \leq b} (|f(x) - g(x)|)$

need R.H.S. set to be bounded above

need $d(f,g) = 0$ iff $f = g$

Proposition Continuous$([a,b], d)$ is a metric space

Standard notation.

$\sup\limits_{a \leq x \leq b} |f(x)| =: \|f\|$ continuous L^∞-norm

L-norm
sup-norm