12-3 products of series

Tuesday, July 9, 2024 1:19 PM

Recall: polynomials of 1 variable

$$P(t) = \sum_{j=0}^{m} a_j t^j \quad [a_j \in \mathbb{C} \text{ or } \mathbb{R}]$$

If $a_m \neq 0$ then $\deg(P) := m$

$$Q(t) = \sum_{j=0}^{n} b_j t^j, \quad b_n \neq 0 \quad \deg(Q) = n$$

clearly $P(t) \cdot Q(t)$ is also a polynomial of degree $\deg(P \cdot Q) = n+m$

$\therefore$ this takes the form

$$P(t) \cdot Q(t) = a_m b_n t^{m+n} + (\text{lower order terms})$$

Let's write

$$(PQ)(t) = \sum_{j=0}^{m+n} c_j \cdot t^j$$

$$\boxed{\text{Claim: } c_j = \sum_{k+i=j} a_k b_i} \longleftarrow \text{Prove this.}$$

Next: let $\{a_n\}_{n\geq 1}$ & $\{b_n\}_{n\geq 1}$ be two infinite sequences

define: for each $n \geq 1$

$$c_n := \sum_{i+j=n} a_i b_j$$

Then the formal product of $\left(\sum_{n\geq 1}^{\infty} a_n\right)\left(\sum_{n\geq 1}^{\infty} b_n\right) =$

$$= \sum_{n\geq 1}^{\infty} c_n$$

Main Theorem

$\sum |a_n| = \sum a_n$

A is a set of convergent values.

Assume: ① $\sum_{n=0}^{\infty} a_n$ converges absolutely (set $A := \sum_{n=0}^{\infty} a_n$)

② $\sum_{n=0}^{\infty} b_n$ converges absolutely or simply converges

Then $\sum_{n\geq 0} c_n$ converges & is equal to

$$\left(\sum_{n\geq 0} a_n\right) \cdot \left(\sum_{n\geq 0} b_n\right) = \sum_{n\geq 0}^{\infty} c_n$$