Lec. 1 - part B Tuesday, June 18, 2024 8:56 AM Bijection then X & X have the same conductify €> 7 bijectum f: X → Y Canton's Diagonalization Process Claim: To = {1,23 .- 3 & Dro
have the Same cardinality. 3 4 - - - -There are an $\infty H ob Q$ between 1 and 2 froof at chim Exhibit Q>0 in an infinite gold $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}$ funce f(1) f(3) f(4) f(5) f(4) f(5) f(4) f(5) fEmousing out redundancies This gives an onto map from Z>0 -> Q>0 Def ! " A set S' in bijection @ Z is called countable set Mrk: Assume A & B Che soft - both f. n. te $A = \{a_1, a_2, a_3, \dots, a_m\}$ B = { b, , b2 , 63 ... bn} thon 9 bijection f: A -> B iff n=m & Cardinality of A (finite) the A:=m froof of Clark Let X be a set (+0) let R be a relation on X RC XXX Defin A allitin R is Symmetric provided (9,b) € R € (b,a) € R Deft Ax celebration Reflexive provided (a,a)ER facx in Perticular Dan (P) = X DOFF : Ar celled is transitive provided $(a,b) \in R \neq (a,c) \in R \Rightarrow (a,c) \in R$ Doff first in a sot X of called in equivilence redation R is Lymnetriz, Reflexive, transfive Gren en equipilence reporten R We reliably write and to man Pa & R-equilent to b Confound N & R Given ~ on an eft volut, monecupty · let u-t X petine the equivilue abst denoted by [a]: TaI := {xt X | x~a} Note: [a] = X Let X= To Detine on Equipinar Pelaphian Z Claim R is reflexive, symm., tras froot at clan 2 [0] = [12] = [206] Emez (m, o) ER (=>m-0 15 ever(=>)

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ever) Wext, let X be a Let & relay. They we dethe X/~ ("X rod tilda") The grotint spre's

of x by the Ed Cela.)

X/~:= {[a] | a { X}}

albert x

An If S M any Set. then the power set of S. F(S) 10 P(s)= SA | A = 53 Leppole S={1,2,3,....n} Than PCS) is finite Q? what is IPCS)? 2151 X=Z & man, A= m-n is even Z/~={[0],[1]} ever odd proposifyn: let u, beX Then offer 1.) [A]=[b] (and 2,) [a]n[b]= p If the Luber TaJE [6] are distinct (cl 3 xell st. xela) Then they are disgint of prove claim A collection e:= {P. PEX3 II Said to be pairwise disjoint provided + P. Q & C & P & Q $A = \{1, 2, 3, 4, 5, 6, 7\}$ notice $C = \{\{1, 3, 43, \{2, 7\}\}, \{5, 3\}\}$ out A Partition C of X is a collecter of per-coupty lasser PCX onhose union is x Propositn: Let ~ be an EQ an a Set X Then $\chi/\omega = \{ [a] | a \in \chi \}$ is a farteton of X B I Define a maj $\mathbb{T}: \mathbb{X} \to \mathbb{X}/\!\!\!/$ y a + > T(a):=[a] where Tits anto (by det of X/~) Hence on ERN on a set X gus set X/~ and map on anto the Cartinued -- Comversely: let f:X->Y
onte se f(x)=Y « Define an EQ~1 m/ S.t. a ~ + 6 () = f(a) = f(b) then of 15 replease symm transition This the existence of X/24 Proparity a) I bijedus beton Ymd X/14 b.) lot a EX HONEGI = f (Eta) 3) The Bibe we for Notion: onto maps, equivilege relating And forfothers hold the Same Propert - Sure They he eferiful acets C(Q):=\(\frac{1}{5}\): \(\frac{1}{10}\) SYKENIO => INEZZO S.E. the different ton, m>N Thus an denset for eld is called a couchy sequence of rattanals list cannal HS +(1), f(2), f(3) ... f(1) = f(2) 13 okay, Remork: Let & be any fet is a softence of elements of x $\chi_i = f(i), \quad \chi_2 = f(2), \quad \chi_3 = f(3), dx$ West Define on equivilence relation ~ on C(D) as fallows. f~ g => YKt Zx 7W $SE * n > N [f(n) - g(n)] < \frac{1}{\kappa}$ Class! ~ U sell on equivere relate (fa), fa), fa) f(n)....) (g(i),g(2),g(3)....-g(n)...) in the limit for) = gar) f(x) = f(x)The Reals