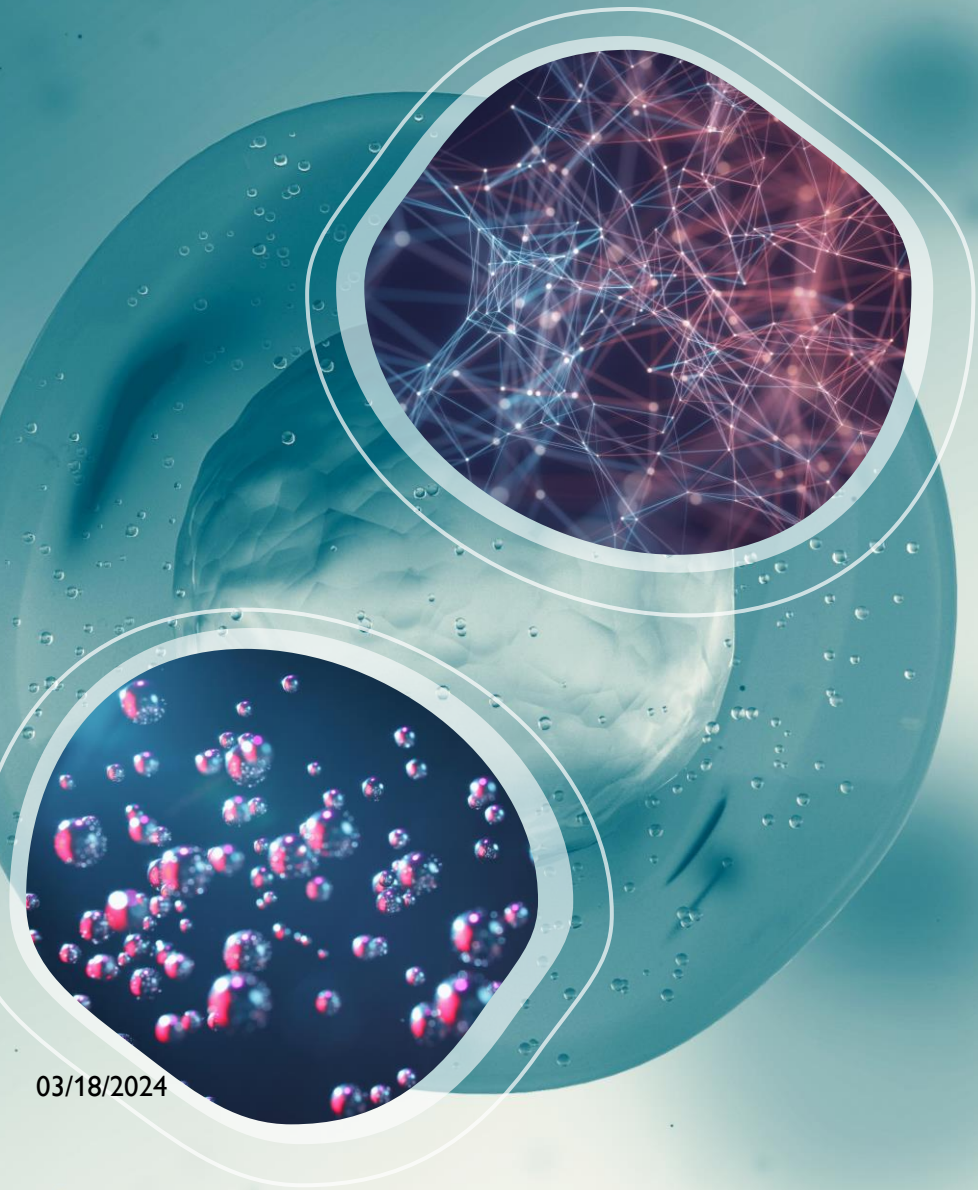
Subspace Topology

Shia Aaron Lloyd Fisher



03/18/2024

Agenda

- Motivation
- Criteria
- Axioms of Topology Review
- Examples
- Validity (Demo)
- Additional Properties
- References

Motivation

- **We want to:**
  - **Consider subsets of a given topological space as its own topological space.**
  - **Determine other properties or implications result from the new topology**


$a^0 = 1\ [a\,0]$
arcsi

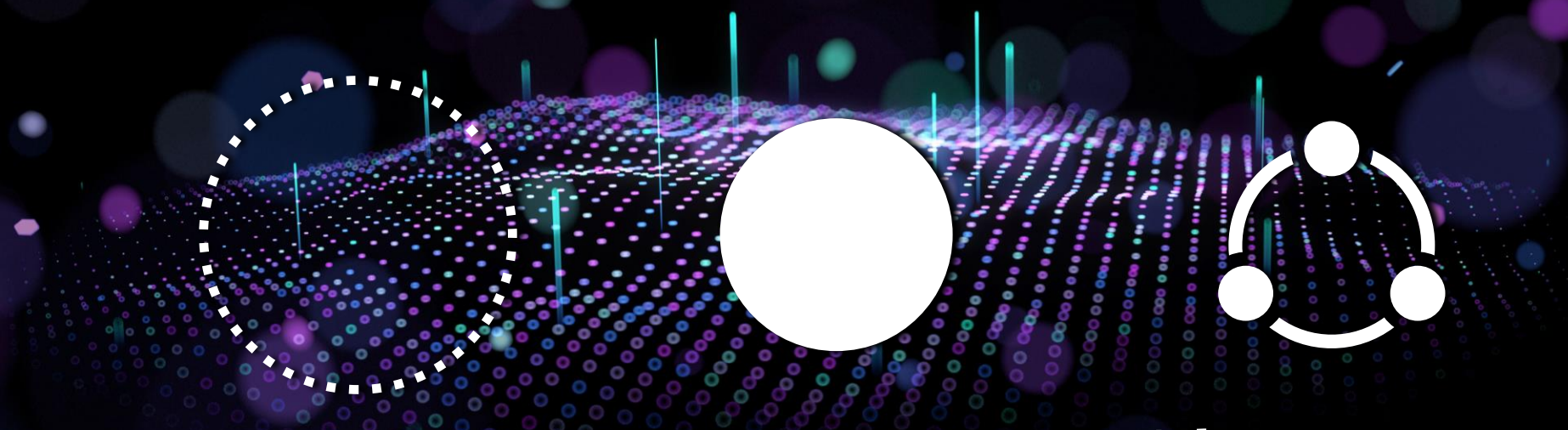
Criteria

(X, τ) $y \subset x$

u is a member of x

$\tau_y := \{ y \cap u \mid u \in \tau \}$

Axioms of a Topology



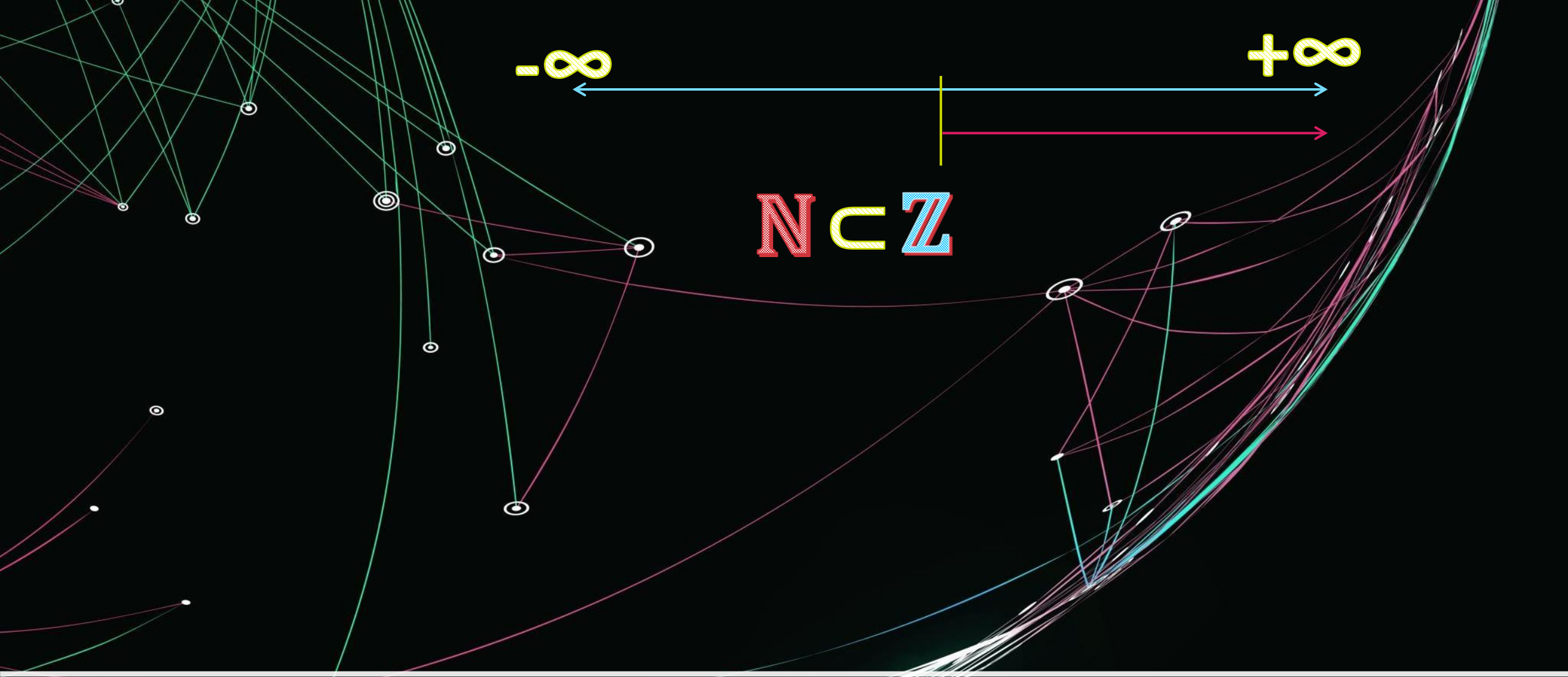
The empty set is open in the space, $\{\} \in \tau$

If X is a topology, then the entire set X is open in τ.

(X, τ)

Arbitrary unions of X are open in τ. For all open members of U in X, $\bigcup_{i=0}^{\infty} U_i$ belongs to τ

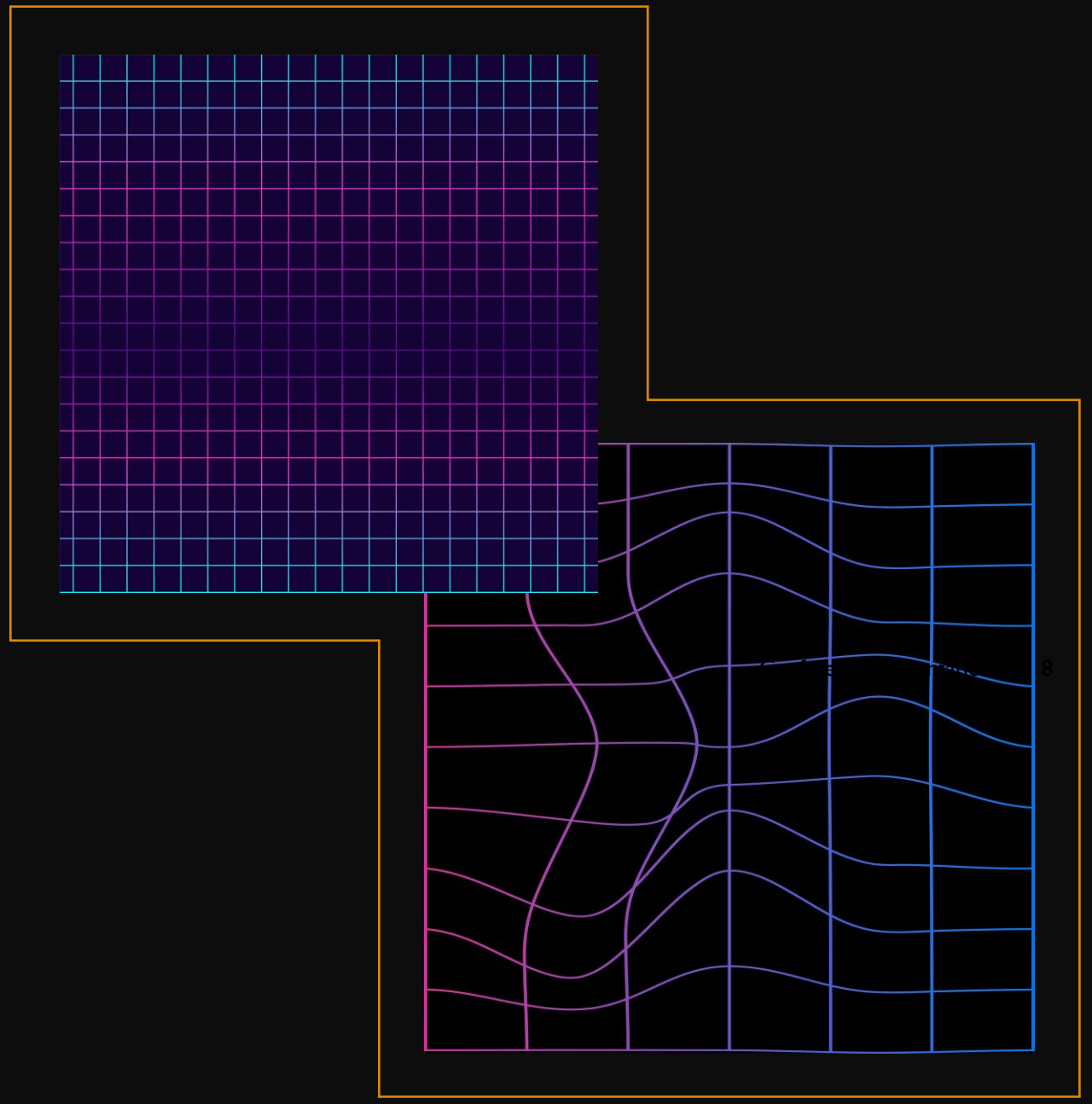
Arbitrary intersections of X are open in τ. For all open members of V in X, $\bigcap_{i=0}^{\infty} V_i$ belongs to τ



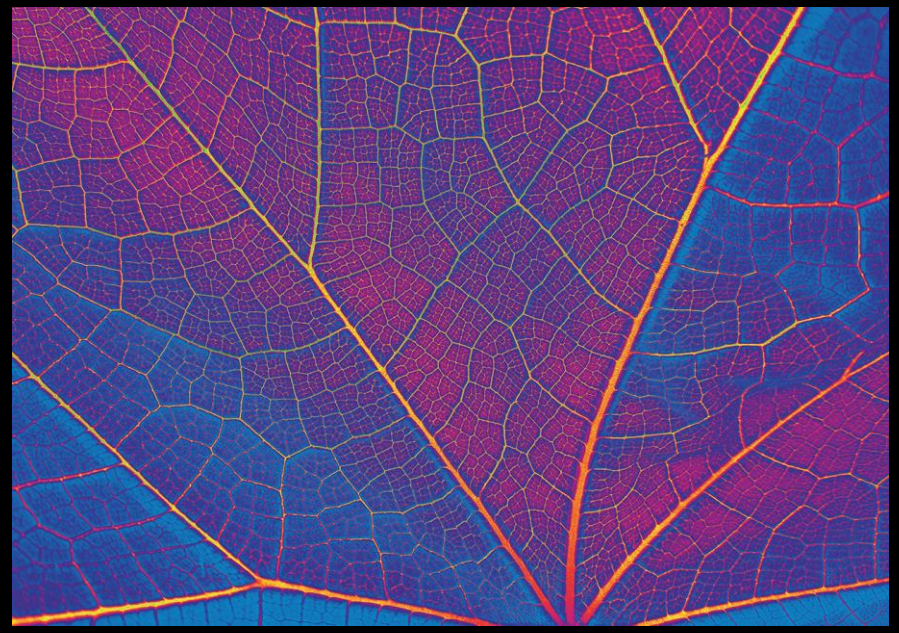
Examples with Sets of Numbers

Validity

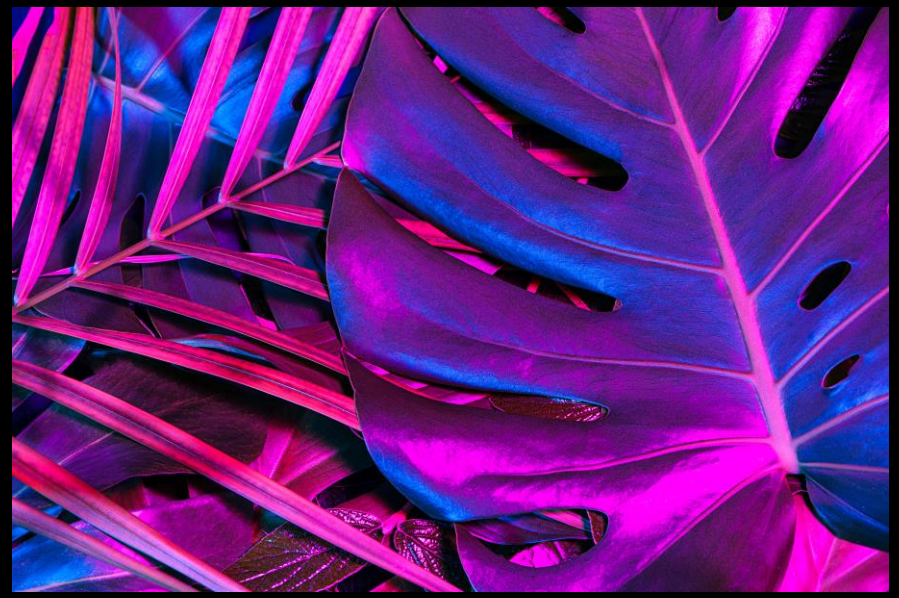
- $\{\}, Y \in \tau_Y$
- Arbitrary unions of Y belong to τ
- Arbitrary intersections of Y belong to τ



Validity continued



Other properties that arise . . .



I. All open sets and all closed sets of a *metrizable* space are implies subspaces are also metrizable.

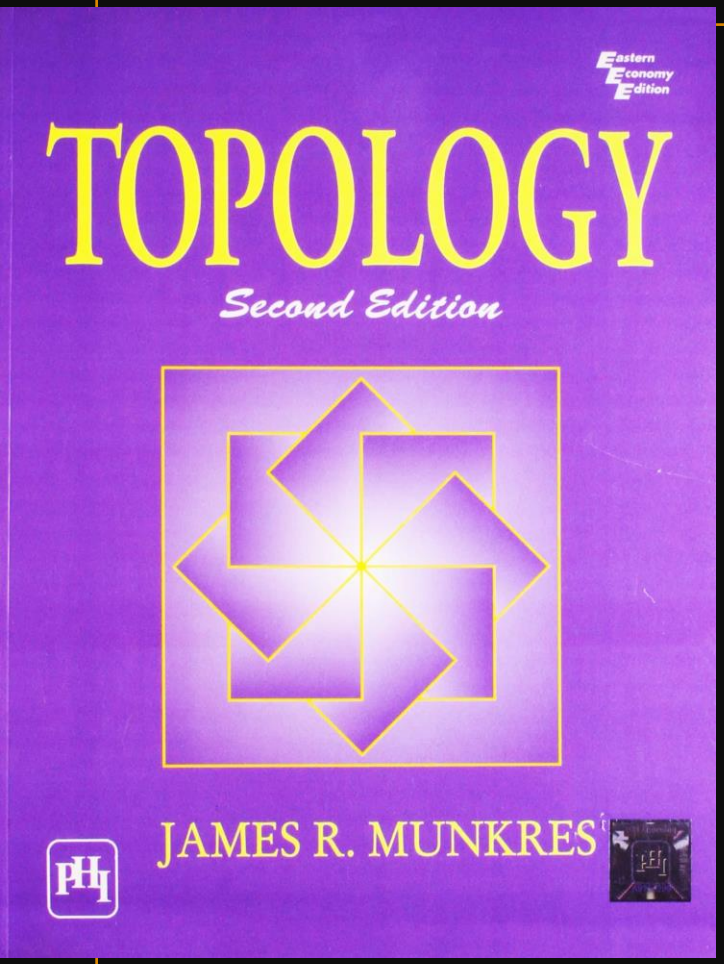
II. It follows, *Hausdorff* spaces imply their subspaces are also Hausdorff. (Theorem 17.11 & 31.2)

III. If $\mathcal{B}$ is a *Basis* of X, then $\mathcal{B}_Y$ is a Basis of the for the subspace of Y defined by:

$$\{B \cap Y \mid B \in \mathcal{B}\}$$

(Lemma 16.1)

References



Munkres, James R., 1930-. (2000). Topology. Upper Saddle River, NJ :Prentice Hall, Inc.,

Thank you

- Shia Aaron Loyd Fisher
- 608-347-7100
- safisher3@wisc.edu
- https://DailyShia.com/