# 4.1 Group Actions and permutation representations

Thursday, November 2, 2023 11:56 AM

recall a group action is defined by

$(G, \circledast)$ a group action is a map

$a \circledast G = ag \quad \forall a \in A, \ \forall g \in G$

$G \times A \to A$

$(g, a) \mapsto g \cdot a$

where

$\sigma_g : A \to A$

$a \mapsto g \cdot a$

So

$g_1, g_2 \in G$

$a \circledast (g_1 g_2) = (a g_1) g_2$

And $a \circledast 1_g = a$

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Recall the Kernel of an Action

$Ker(a) = \cancel{\{ a \in A \mid ag = 1_g \} \ \forall g \in G}$

$\{ g \in G \mid g \cdot a = a \} \ \forall a \in A$

Recall the permutation representation

$\sigma_g(a) : 1_{S_A} \qquad \forall g \in G$

$\{ g \in G \mid \sigma_g = 1_{S_A} \}$

$= \{ g \in G \mid \sigma_g(a) = 1_{S_A}(a) \ \forall a \in A \}$

$$= \{ g \in G \mid g \circ a = a, \ \forall a \in A \}$$

**Ex 3.** (a) Find all elts of $A_3 = \{1, (1\ 2\ 3), (1\ 3\ 2)\}$

(b) Write all elts of the set $\mathbb{F}_2^3 = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \ x_1, x_2, x_3 \in \mathbb{F}_2 = \{0,1\} \right\}$ in the row $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. I'll give instructions for the rest of the table in class.

| label | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ | $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ | $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ | $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ | $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ | $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ | $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ | $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ | $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ |
| $(1\,2\,3)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_1 \\ x_2 \end{bmatrix}$ | | | | | | | | |
| $(1\,3\,2)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ | | | | | | | | |

$$(1\ 2\ 3)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \ \end{bmatrix}$$