2.4 Subgroup Generated by Subsets of a Group

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recall

Let G be a group $A \subseteq G$ a subset

the smallest subgroup G containing A

Idea 1

$$\left\{ a_1^{\varepsilon_1}, a_2^{\varepsilon_2} \ldots a_n^{\varepsilon_n} \;\middle|\; n \in \mathbb{Z}_{\geq 0},\ a_i \in A,\ \varepsilon_i \in \{1,-1\} \text{ for each } i \right\}$$

Words whose alphabet are elements of A

Indeed a subgroup of G containing A

Idea 2

$$J = \bigcap_{\substack{H \leq G \\ A \subseteq G}} H$$

the intersection of all the sub groups H of G

H containing A

$H_1 \cap H_2 \cap \ldots$

$$\bigcap_{i=1}^{\infty} H_i$$

subgroup criteria

if $x, y \in J$ is $xy^{-1} \in J$

Let $x, y \in J$ then $x, y \in H$,

~~$\partial : J \to J$~~

~~$\partial_H(j) \circ \partial_H^{-1}(j)$~~

for $\underline{\gamma_G = x \quad (\gamma_G)^{-1} = y}$

because $1 \in H$ for

$= J$

all $H \leq G$ and $A \subseteq H$

so

Suppose K is also a subgroup of G containing A then $J \subseteq K$

Because a group by definition has an identity and an inverse

$C_G(A) = \{ g \in G \mid gag^{-1} = a \ \forall a \in A \}$

$\Rightarrow N_H(a) = \{ \quad \}$

"$\subseteq$" If $x \in LHS$ then $x \in H$ for any subgroup H of G containing A

so $x \in RHS$

"$\supseteq$" Note that the LHS is a subgroup of G containing A

if $y \in RHS$ then $y \in$ subgroup of G in particular $y \in LHS$